Passmores Academy Mathematics Department

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Year 6 into 7 Mathematics Project

Who invented multiplication and how many different forms are there?

Contents

Introduction

Multiplication was invented by ancient Babylonians. It is a type of mathematics that developed over time and no one person invented it. To get you started here is an article describing 25+ Ways to Multiply. (http://threesixty360.wordpress.com/25-ways-to-multiply/)

- **Repeated Addition**
- **On the fly shortcuts** to Addition
- **Doubling and Halving** (Egyptian, Ethiopian?, Russian?)
- **Doubling and Adding** or Duplation (Egyptian)
- **Shift and Add**
- **Grid or Lattice Multiplication** (Arabic, Indian?)
- **A variation** on Grid Multiplication (medieval Italian or earlier)
- **"Traditional"** or Long Multiplication (medieval Italian or earlier)
- **Crocetta**, or Vertically and Crosswise Multiplication (Indian)
- **Digit-reverse and shift**, by Pappus (Greek)
- **The Method of the Cups** (Spain or the Americas?)
- ***per Repiego***, or multiplying by factors (medieval Italy or earlier)

Formulas and Tables:

- **Babylonian**(?) (First Formula)
- **Babylonian**(?) (Second Formula)
- **A third difference of squares** (Greek)
- **Prosthaphaeresis** or Trig Tables (Europe)
- **Logarithm Tables** (Europe)

Physical Objects:

- **Napier's Rods** (Scotland)
- **Genaille-Lucas Rulers** (France)
- **Abacus** (Chinese)
- **"Prosthaphaeretic" Slide Rule** (based on similar triangles)

Other Methods:

- **Similar Triangles** (Greek)
- **Finger Multiplication by 9** and other multiplication tricks for multiplying by a specific digit
- **Finger Multiplication between {5, 6, 7, 8, 9, or 10}** (medieval Europe?)
- **Finger Multiplication between {10, 11, 12, 13, 14, or 15}** (medieval Europe?)

There are lots of other methods but this list should get you started.

Your task:

Your task is to research and show you can use different methods of multiplication.

First, you need to decide on your level of challenge. Go on, you know you want to go for platinum!

PLATINUM

Platinum:

In order to achieve platinum, you need to create an instructional booklet that your fellow classmates could use to learn new methods of multiplication.

Your instructional booklet must include:

- At least five methods that are not included on the list on page 2. Use the internet to help you find new methods.
- Do a fully worked example of each of the methods you chose.
- Use the worked example to describe how the method works (give students steps they can follow to try it themselves)
- Explain the advantages and disadvantages of using each these methods.

Then, using **one** of the methods you found, answer the following questions:

1. 1294 x 531

2. 4868 x 842

3. 20.74 x 62

4. 79.8 x 0.9

5. Find the area of a rectangle when the length is 67.8cm and the width is 14.2cm.

6. A group of 862 people go to the cinema. 492 of these are children. The ticket price for an adult is £8.59 each. The ticket price for a child is £3.25. Work out the total cost for the group.

GOLD

Gold:

To achieve gold, you must complete the following:

- Use the list of the 25 methods of page 2 to identify five methods that you haven't used (yet!)
- In your own words, describe how you do each of the five methods that you chose.
- Do a worked example of each of the five methods.
- Explain the advantages and disadvantages of using each of the five methods.
- Using **one** of the methods, answer the following questions:

1. 345 x 87

2. 9478 x 45

3. 386 x 927

4. 84.5 x 9

5. 3.49 x 8.2

6. A group of 42 people go to the cinema. The ticket price is £8.59 each. Work out the total cost for the group.

7. Work out the area of a square with side length 5.7m.

SILVER

Silver:

To achieve silver, you must complete the following:

- Use the list of the 25 methods on page 2 to identify three methods that you haven't used (yet!)
- In your own words, describe each of the three methods that you chose.
- Do a worked example of each of the three methods that you chose
- Using **one** of the methods, answer the following questions:

1. 34 x 87

2. 435 x 78

3. 341 x 967

4. 4938 x 87

5. 3.49 x 87

6. 1.9 x 9.6

BRONZE

Bronze:

To achieve bronze, you must complete the following:

- Choose **two** of the five methods from the next page.
- In your own words, write a short explanation of how the two methods work.
- Using **one** of the methods, answer the following questions:

1. 54 x 9

2. 34 x 78

3. 486 x 8

4. 456 x 87

5. 932 x 89

6. 19 x 875

A. The Grid Method was known to the Ancient Greeks, and is included in Euclid's Elements. Use a computer to see the example:
http://wsgfl2.westsussex.gov.uk/Aplaws/maths/multicultural/Docs/MAWGrids.doc

B. The Egyptian Method. Based on doubling, this ancient method is included in the Rhind Papyrus (effectively an Egyptian mathematics 'text book') in the British Museum. Use a computer to see the example: http://wsgfl2.westsussex.gov.uk/Aplaws/maths/multicultural/MAWEgypt.htm

C. The Russian Peasant Method has some similarities with the Egyptian method. The method has a very pleasing 'Wow!' factor, as it seems to rely on sloppy working and the correct answer seems to appear by magic! Use a computer to see the example:
http://wsgfl2.westsussex.gov.uk/Aplaws/maths/multicultural/MAWRuss.htm

D. Gelosia is a method imported (like so many other mathematical ideas) into Europe from India in the middle ages. The present name for the method derives from a type of Venetian window frame! Use a computer to see the example:
http://wsgfl2.westsussex.gov.uk/Aplaws/maths/multicultural/MAWNB.htm

E. Napier's Bones are a technological development of the Gelosia method, developed by John Napier (1550 – 1617) of Scotland, who also developed a system of logarithms and the notation for the decimal point. The products needed in the Gelosia method were engraved on strips of wood or bone, and the answer could be found by arranging the strips appropriately. Sets of Napier's Bones were usually fixed to each face of a square-sectioned rod, allowing several repeated strips in a set. Use a computer to see the example:
http://wsgfl2.westsussex.gov.uk/Aplaws/maths/multicultural/MAWNB.htm

Each link consists of a summary of the method, based on a common example. There are brief notes on the working of the method, (http://wsgfl2.westsussex.gov.uk/Aplaws/maths/multicultural/MAWIntro.htm)

The Grid Method

X	100	30	4	Row total
30	30x100=3000	30x30=900	30x4=120	4020
9	9x100=900	9x30=270	9x4=36	1206
Column total	3900	1170	156	Solution: 5226

How it works

There are many variations upon this basic theme, but they are all based on the idea of finding the area of a rectangle. The rectangle does *not* need to be drawn to scale. Each number to be multiplied is considered to be the length of the side of a rectangle, and the calculation reduces to the task of finding the area of the rectangle. The partial products are written inside the rectangle, as shown, and they then need to be added up.

The Egyptian Method

Example: 134×39

$1 \times 39 = 39$

$2 \times 39 = 78$

$4 \times 39 = 156$

$8 \times 39 = 312$

$16 \times 39 = 624$

$32 \times 39 = 1248$

$64 \times 39 = 2496$

$128 \times 39 = 4992$

$134 = 128 + 4 + 2$

So, $134 \times 39 = (128 + 4 + 2) \times 39 = 128 \times 39 + 4 \times 39 + 2 \times 39$

$= 4992 + 156 + 78 = 5226$

How it works

Take either one of the numbers being multiplied, and form a table by successive doubling as shown above. In the example we stop at 128×39, because the next doubling would give 256×39, which is too big. Next, we find out how to make up 134 by adding powers of 2. To do this, simply take away the biggest power of 2 that goes into 134: $134 - 128 = 6$. Now continue with the 6. The biggest power of 2 that fits is 4, leaving 2, which is a power of 2. So, $134 = 128 + 4 + 2$. Now find the numbers corresponding to 128, 4, and 2, which are 4992, 156 and 78. Add these up to give the answer of 5226.

The Russian Peasant Method

Example: 134 × 39

~~134~~	~~39~~
67	78
33	156
~~16~~	~~312~~
~~8~~	~~624~~
~~4~~	~~1248~~
~~2~~	~~2496~~
1	4992

Answer = total of the numbers not crossed out in the right-hand column = 78 + 156 + 4992 = 5226

How it works

Start by writing down the two numbers to be multiplied, side-by-side.

Now halve the number on the left and write the answer underneath it.

Then double the number on the right and write that answer underneath.

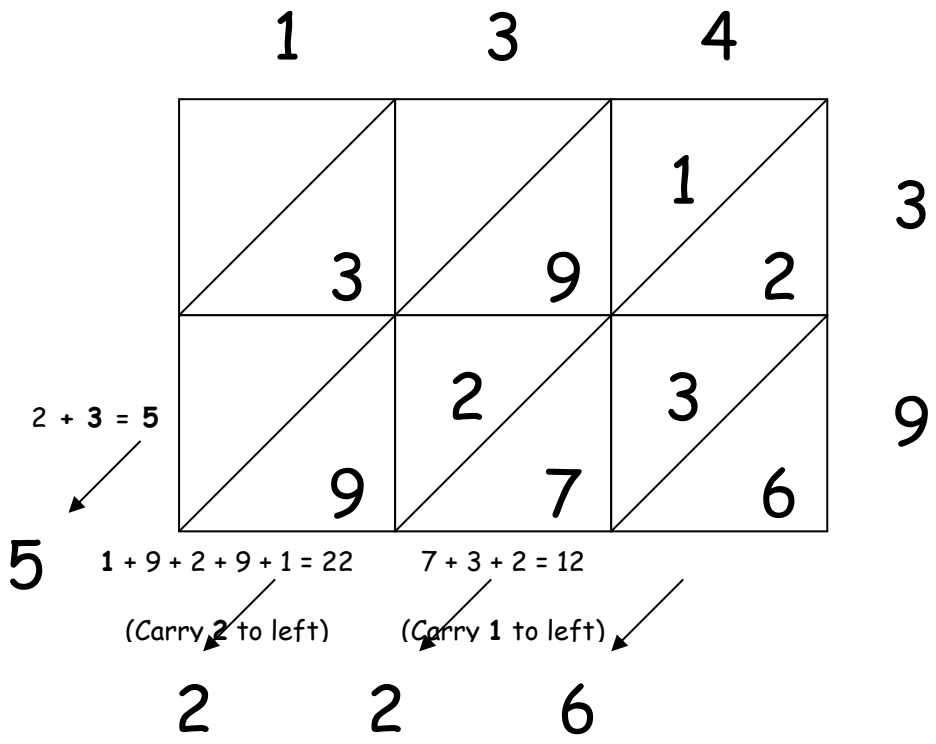
We then carry on down the columns, halving the numbers on the left and doubling the numbers on the right – but with just one little twist…

Halving the left-hand number is easy when that number is even, but what if it's odd? Couldn't be easier – just 'throw the half away'! Half of 67 is 33!

Keep working down the columns until the number on the left is 1, then stop. Now cross out each row of the table where the number on the *left* is *even*. Finally, add up the 'non-crossed-out' numbers on the right to get the answer.

Gelosia

Example: 134 × 39



How it works

Rather like the grid method, we start with a rectangle with numbers marked on the sides. In Gelosia, however, the rectangle is divided up into squares as shown, and one digit of each of the numbers being multiplied is put at the top of each column, or end of each row, of squares. Each square is split diagonally, and each 'partial product' is written into its square with the tens digit above the line and the units digit below. To find the answer, start from the right and add along each diagonal; carry any 'tens' digits over to the next diagonal. Finally, read the answer left-to-right as usual!

Napier's Bones

Example: 134 × 39

*	1	3	4
1	1	3	4
2	2	6	8
3	3	9	12
4	4	12	16
5	5	15	20
6	6	18	24
7	7	21	28
8	8	24	32
9	9	27	36

We use the strips with 1, 3 and 4 at the top.

Line them up as shown, placing the special 'key' strip (with the star at the top) on the left.

Now, 134 × 30 = 134 × 3 × 10, and we can read the answer to 134 × 3 from the strips; just add down the diagonals in the row with '3' at the start, starting at the right. There will always be just 2 digits to add. Carry to the left if necessary.

So, 134 × 30 = 402 × 10 = 4020.
Next, 134 × 9 = 1206.
Finally we add the two partial products: 134 × 39 = 4020 + 1206 = 5226.

How it works

This is a technological approach to multiplication; there is no need to learn tables, or any arithmetic skills beyond addition and simple partitioning. Each 'box' on a strip shows the result of multiplying the top digit by the corresponding digit on the 'key' strip; the answer is split into its contribution to the current place value column and the next one.

References

(n.d.). Retrieved from http://wsgfl2.westsussex.gov.uk/Aplaws/maths/multicultural/MAWIntro.htm.

http://threesixty360.wordpress.com/25-ways-to-multiply/. (n.d.). Retrieved from http://threesixty360.wordpress.com/25-ways-to-multiply/.