

**GCSE Mathematics**

**Scheme of Work - Higher**

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| **1. Number**  |

**KEYWORDS**

Integer, number, digit, negative, decimal, addition, subtraction, multiplication, division, remainder, operation, estimate, power, roots, factor, multiple, primes, square, cube, even, odd, surd, rational, irrational standard form, simplify, rationalise, denominator, fraction, recurring, product, powers, indices, roots

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| **1a. Recurring fractions**  | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to:

Find equivalent fractions and compare the size of fractions;

Add, subtract, multiply and divide fractions;

Multiply and divide fractions, including mixed numbers and whole numbers and vice versa;

Add and subtract fractions, including mixed numbers;

Understand and use unit fractions as multiplicative inverses;

Find the reciprocal of an integer, decimal or fraction.

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Convert a fraction to a decimal to make a calculation easier;
* By writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals;
* Convert a fraction to a recurring decimal;
* Convert a recurring decimal to a fraction;

**POSSIBLE SUCCESS CRITERIA**

Prove whether a fraction is terminating or recurring.

Convert a fraction to a decimal including where the fraction is greater than 1.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

**COMMON MISCONCEPTIONS**

The larger the denominator, the larger the fraction.

**NOTES**

Recognise that every terminating decimal has its fraction with a 2 and/or 5 as a common factor in the denominator.

Use long division to illustrate recurring decimals.

Encourage use of the fraction button.

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| **1b. Fractional & Negative Indices**  | **15Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to:

Use index laws to simplify and calculate the value of numerical and algebraic expressions involving multiplication and division of integer powers, fractions and powers of a power;

Use numbers raised to the power zero, including the zero power of 10;

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Use index notation for integer powers of 10, including negative powers;
* Estimate powers and roots of any given positive number, by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
* Find the value of calculations using indices including positive, fractional and negative indices;
* Recall that *n*0 = 1 and *n*–1 =  for positive integers n as well as,  = √*n* and  = 3√*n* for any positive number *n*;
* Understand that the inverse operation of raising a positive number to a power *n* is raising the result of this operation to the power ;
* Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractional and negative powers, and powers of a power;
* Solve problems using index laws;
* Use brackets and the hierarchy of operations up to and including with powers and roots inside the brackets, or raising brackets to powers or taking roots of brackets;
* Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.

**POSSIBLE SUCCESS CRITERIA**

Prove that the square root of 45 lies between 6 and 7.

Evaluate (23 × 25) ÷ 24, 40, .

Work out the value of *n* in 40 = 5 × 2*n*.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of Mathematics.

**COMMON MISCONCEPTIONS**

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

**NOTES**

Students need to know how to enter negative numbers into their calculator.

Use negative number and not minus number to avoid confusion with calculations.

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| **1c. Product Rule**  | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to:

Add, subtract, multiply and divide decimals and whole numbers;

Multiply or divide by any number between 0 and 1;

Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;

Round numbers to the nearest 10, 100, 1000;

Round to the nearest integer, to a given number of decimal places and to a given number of significant figures;

Estimate answers to one- or two-step calculations, including use of rounding numbers and formal estimation to 1 significant figure: mainly whole numbers and then decimals.

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Use the product rule for counting (i.e. if there are *m* ways of doing one task and for each of these, there are *n* ways of doing another task, then the total number of ways the two tasks can be done is *m* × *n* ways);

**POSSIBLE SUCCESS CRITERIA**

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

**COMMON MISCONCEPTIONS**

**NOTES**

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| **1d. Accuracy and bounds**  | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to :

Substitute numbers into an equation and give answers to an appropriate degree of accuracy.

Use inequality notation to specify an error interval due to truncation or rounding.

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Calculate the upper and lowers bounds of numbers given to varying degrees of accuracy;
* Calculate the upper and lower bounds of an expression involving the four operations;
* Find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy;
* Find the upper and lower bounds of calculations involving perimeters, areas and volumes of 2D and 3D shapes;
* Calculate the upper and lower bounds of calculations, particularly when working with measurements;

**POSSIBLE SUCCESS CRITERIA**

Work out the upper and lower bounds of a formula where all terms are given to 1 decimal place.

Be able to justify that measurements to the nearest whole unit may be inaccurate by up to one half in either direction.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

This sub-unit provides many opportunities for students to evaluate their answers and provide counter-arguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

**COMMON MISCONCEPTIONS**

Students readily accept the rounding for lower bounds, but take some convincing in relation to upper bounds.

**NOTES**

Students should use ‘half a unit above’ and ‘half a unit below’ to find upper and lower bounds.

Encourage use a number line when introducing the concept.

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| **1e. Surds** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able t:

Use negative numbers with all four operations.

Recall and use the hierarchy of operations.

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Understand surd notation, e.g. calculator gives answer to sq rt 8 as 4 rt 2;
* Simplify surd expressions involving squares (e.g. √12 = √(4 × 3) = √4 × √3 = 2√3).
* Rationalise the denominator involving surds

**POSSIBLE SUCCESS CRITERIA**

Simplify √8.

Rationalise: , , (√18 + 10) +√2.

Explain the difference between rational and irrational numbers.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Links with other areas of Mathematics can be made by using surds in Pythagoras and when using trigonometric ratios.

**COMMON MISCONCEPTIONS**

√3 x √3 = 9 is often seen.

**NOTES**

Revise the difference of two squares to show why we use, for example, (√3 – 2) as the multiplier to rationalise (√3 + 2).

Link collecting like terms to simplifying surds (Core 1 textbooks are a good source for additional work in relation to simplifying surds).

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| **2. Algebra**  |

**KEYWORDS**

Expression, identity, equation, formula, substitute, term, ‘like’ terms, index, power, negative and fractional indices, collect, substitute, expand, bracket, factor, factorise, quadratic, linear, simplify, approximate, arithmetic, geometric, function, sequence, *n*th term, derive, Rationalise, denominator, surd, rational, irrational, fraction, equation, rearrange, subject, proof, function notation, inverse, evaluate

**PRIOR KNOWLEDGE**

Students should be able to:

Use algebraic notation and symbols correctly;

Write an expression;

Know the difference between a term, expression, equation, formula and an identity;

Manipulate an expression by collecting like terms;

Substitute positive and negative numbers into expressions such as 3*x* + 4 and 2*x*3 and then into expressions involving brackets and powers;

Substitute numbers into formulae from mathematics and other subject using simple linear formulae, e.g. *l* × *w*, *v* = *u* + *at*;

Simplify expressions by cancelling, e.g.  = 2*x*

Use instances of index laws for positive integer powers;

Use index notation (positive powers) when multiplying or dividing algebraic terms;

Multiply a single term over a bracket;

Recognise factors of algebraic terms involving single brackets and simplify expressions by factorising, including subsequently collecting like terms;

Know that squaring a linear expression is the same as expanding double brackets

Rearrange simple equations to change the subject

Simplify surds.

Use negative numbers with all four operations.

Recall and use the hierarchy of operations.

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| **2a. Expanding & factorising**  | **Teaching time**TBC |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Expand the product of two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to (2*x* + 3*y*)(3*x* – *y*);
* Expand the product of more than two linear expressions;
* Factorise quadratic expressions of the form *ax*2 + *bx* + *c*;
* Factorise quadratic expressions using the difference of two squares.

**POSSIBLE SUCCESS CRITERIA**

Expand and simplify (3*x* + 2)(4*x* – 1).

Factorise 6*x*2 – 7*x* + 1.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

**COMMON MISCONCEPTIONS**

When expanding two linear expressions, poor number skills involving negatives and times tables will become evident.

**NOTES**

Plenty of practice should be given for factorising, and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed. Encourage students to expand linear sequences prior to simplifying when dealing with “double brackets”.

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| **2b. Rearranging equations**  | **Teaching time**TBC |

**OBJECTIVES**

By the end of this sub-unit, students should be able to:

* Change the subject of a formula, including cases where the subject occurs on both sides of the formula, or where a power of the subject appears;
* Change the subject of a formula such as , where all variables are in the denominators;

**POSSIBLE SUCCESS CRITERIA**

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

**COMMON MISCONCEPTIONS**

When simplifying involving factors, students often use the ‘first’ factor that they find and not the LCM.

**NOTES**

Practice factorisation where the factor may involve more than one variable.

Emphasise that, by using the LCM for the denominator, the algebraic manipulation is easier.

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| **2c. Sequences** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to:

Find and use (to generate terms) the *n*th term of an arithmetic sequence;

Use the *n*th term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term above or below a given number;

Identify which terms cannot be in a sequence by finding the *n*th term;

Distinguish between arithmetic and geometric sequences;

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Recognise sequences including those involving numbers in standard form or index form;
* Generate specific terms in a sequence using the position-to-term rule and term-to-term rule;
* Continue a quadratic sequence and use the *n*th term to generate terms;
* Find the *n*th term of quadratic sequences;
* Use finite/infinite and ascending/descending to describe sequences;
* Recognise and use simple geometric progressions (*rn* where *n* is an integer, and *r* is a rational number > 0 or a surd);
* Continue geometric progression and find term to term rule, including negative, fraction and decimal terms;
* Solve problems involving sequences from real life situations.

**POSSIBLE SUCCESS CRITERIA**

Given a sequence, ‘which is the 1st term greater than 50?’

Be able to solve problems involving sequences from real-life situations, such as:

* 1 grain of rice on first square, 2 grains on second, 4 grains on third, etc (geometric progression), or person saves £10 one week, £20 the next, £30 the next, etc;
* What is the amount of money after *x* months saving the same amount, or the height of tree that grows 6 m per year;
* Compare two pocket money options, e.g. same number of £ per week as your age from 5 until 21, or starting with £5 a week aged 5 and increasing by 15% a year until 21.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Evaluate statements about whether or not specific numbers or patterns are in a sequence and justify the reasons.

**COMMON MISCONCEPTIONS**

Students struggle to relate the position of the term to “*n*”.

**NOTES**

Emphasise use of 3*n* meaning 3 x *n*.

Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the *n*th term.

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| **3. Coordinate Geometry**  | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to:

* Identify coordinates of given points in the first quadrant or all four quadrants.
* Use Pythagoras’ Theorem and calculate the area of compound shapes.
* Use and draw conversion graphs for these units.
* Use function machines and inverse operations.
* Identify and plot points in all four quadrants;
* Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills, fixed charge and cost per item;Draw distance–time and velocity–time graphs;
* Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
* Find the coordinates of the midpoint of a line segment with a diagram given and coordinates;
* Calculate the length of a line segment given the coordinates of the end points;
* Find the equation of the line through two given points.
* Calculate an end point of a line segment given one coordinate and its midpoint.
* Plot and draw graphs of *y* = *a*, *x* = *a*, *y* = *x* and *y* = –*x*, drawing and recognising lines parallel to axes, plus *y* = *x* and *y* = –*x*;
* Identify and interpret the gradient of a line segment;
* Recognise that equations of the form *y* = *mx* + *c* correspond to straight-line graphs in the coordinate plane;
* Identify and interpret the gradient and *y*-intercept of a linear graph given by equations of the form *y* = *mx* + *c*;
* Find the equation of a straight line from a graph in the form *y* = *mx* + *c*;
* Plot and draw graphs of straight lines of the form *y* = *mx* + *c* with and without a table of values;
* Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;
* Interpret graphs of quadratic functions from real-life problems;
* Draw graphs of simple cubic functions using tables of values;
* Interpret graphs of simple cubic functions, including finding solutions to cubic equations;
* Draw graphs of the reciprocal function  with *x* ≠ 0 using tables of values;

**KEYWORDS**

Coordinate, axes, 3D, Pythagoras, graph, speed, distance, time, velocity, quadratic, solution, root, function, linear, circle, cubic, approximate, gradient, perpendicular, parallel, equation

**OBJECTIVES**

By the end of the unit, students should be able to (some of which is in the crossover):

* Sketch a graph of a linear function, using the gradient and *y*-intercept (i.e. without a table of values);
* Find the equation of the line through one point with a given gradient;
* Identify and interpret gradient from an equation *ax* + *by* = *c*;
* Find the equation of a straight line from a graph in the form *ax* + *by* = *c*;
* Plot and draw graphs of straight lines in the form *ax* + *by* = *c*;
* Interpret and analyse information presented in a range of linear graphs:
* use gradients to interpret how one variable changes in relation to another;
* find approximate solutions to a linear equation from a graph;
* identify direct proportion from a graph;
* find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
* Explore the gradients of parallel lines and lines perpendicular to each other;
* Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
* Select and use the fact that when *y* = *mx* + *c* is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of *m* and a line perpendicular to this line will have a gradient of .
* Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
* Draw circles, centre the origin, equation *x*2 + *y*2 = *r*2.

**POSSIBLE SUCCESS CRITERIA**

Find the equation of the line passing through two coordinates by calculating the gradient first.

Understand that the form *y* = *mx* + *c* or *ax* + *by* = *c* represents a straight line.

Select and use the correct mathematical techniques to draw linear, quadratic, cubic and reciprocal graphs.

Identify a variety of functions by the shape of the graph.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Given an equation of a line provide a counter argument as to whether or not another equation of a line is parallel or perpendicular to the first line.

Decide if lines are parallel or perpendicular without drawing them and provide reasons.

Match equations of quadratics and cubics with their graphs by recognising the shape or by sketching

**COMMON MISCONCEPTIONS**

Students can find visualisation of a question difficult, especially when dealing with gradients resulting from negative coordinates.

Students struggle with the concept of solutions and what they represent in concrete terms.

**NOTES**

Encourage students to sketch what information they are given in a question – emphasise that it is a sketch.

Careful annotation should be encouraged – it is good practice to label the axes and check that students understand the scales.

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| **4. Surface area & volume – pyramids, cylinders, cones, spheres & frustums** | **Teaching time**TBC |

**MUCH OF THIS UNIT IS BUILT UPON AREA AND VOLUME FROM THE CROSSOVER UNITS – BUT DEVELOPS FURTHER TO INCLUDE FORMING AND SOLVING EQUATIONS IN THIS CONTEXT**

**PRIOR KNOWLEDGE**

Students should:

Recall the names and properties of 3D forms and understand perimeter, area and volume.

Be able to substitute numbers into an equation and give answers to an appropriate degree of accuracy.

Students should know the various metric units.

Be able to find area and perimeters of a triangle, rectangle, trapezium and parallelogram using a variety of metric measures in a straight forward context but also with compound shapes.

Recall the definition of a circle and name and draw parts of a circle; Recall and use formulae for the circumference of a circle and the area enclosed by a circle (using circumference = 2*πr* = *πd* and area of a circle = *πr*2) using a variety of metric measures;

Calculate perimeters and areas of composite shapes made from circles and parts of circles (including semicircles, quarter-circles, combinations of these and also incorporating other polygons);

Calculate arc lengths, angles and areas of sectors of circles;

Find radius or diameter, given area or circumference of circles in a variety of metric measures;

Give answers to an appropriate degree of accuracy or in terms of *π*;

Draw sketches of 3D solids;

Identify planes of symmetry of 3D solids, and sketch planes of symmetry;

Find the surface area of prisms made up of triangles and rectangles, and other shapes with and without a diagram;

Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures;

Use volume to solve problems;

Find the volume and surface area of a cylinder;

Find the surface area of a pyramid;

Use the formulae for volume and surface area of spheres and cones;

Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders;

**KEYWORDS**

Triangle, rectangle, parallelogram, trapezium, area, perimeter, formula, length, width, prism, compound, measurement, polygon, cuboid, volume, nets, isometric, symmetry, vertices, edge, face, circle, segment, arc, sector, cylinder, circumference, radius, diameter, pi, composite, sphere, cone, capacity, hemisphere, segment, frustum, bounds, accuracy, surface area

**OBJECTIVES**

By the end of the unit, (in addition to the prior knowledge above) students should be able to:

* Convert between metric units of length, area and volume.
* Calculate the maximum and minimum values of measurements and calculations;
* Solve problems involving surface area and volume for more complex shapes and solids, including segments of circles and frustums of cones;
* Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders;
* Form equations involving more complex shapes and solve these equations.
* Give answers to an appropriate degree of accuracy or in terms of *π*;

**POSSIBLE SUCCESS CRITERIA**

Given two solids with the same volume and the dimensions of one, write and solve an equation in terms of *π* to find the dimensions of the other, e.g. a sphere is melted down to make ball bearings of a given radius, how many will it make?

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Multi-step problems, including the requirement to form and solve equations, provide links with other areas of mathematics.

**COMMON MISCONCEPTIONS**

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**NOTES**

Encourage students to draw a sketch where one isn’t provided.

Emphasise the functional elements with carpets, tiles for walls, boxes in a larger box, etc. Best value and minimum cost can be incorporated too.

Ensure that examples use different metric units of length, including decimals.

Ensure that students know it is more accurate to leave answers in terms of *π*, but only when asked to do so.

Formulae for curved surface area and volume of a sphere, and surface area and volume of a cone will be given on the formulae page of the examinations.

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| **5. Transformations**  | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to

Recognise 2D shapes.

Plot coordinates in four quadrants and linear equations parallel to the coordinate axes.

Enlarge, rotate, reflect and translate given shapes and also be able to describe transformations using the appropriate terminology.

**KEYWORDS**

Rotation, reflection, translation, transformation, enlargement, scale factor, vector, centre, angle, direction, mirror line, centre of enlargement, describe, distance, congruence, similar, combinations, single, corresponding, constructions, compasses, protractor, bisector, bisect, line segment, perpendicular, loci, bearing

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Distinguish properties that are preserved under particular transformations and combinations of transformations;
* Understand the effect of one translation followed by another, in terms of column vectors (to introduce vectors in a concrete way);
* Find areas after enlargement and compare with before enlargement, to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);
* Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations;
* Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements;
* Describe the changes and invariance achieved by combinations of rotations, reflections and translations.
* Describe and transform 2D shapes using enlargements by negative scale factor;

**POSSIBLE SUCCESS CRITERIA**

Recognise similar shapes because they have equal corresponding angles and/or sides scaled up in same ratio.

Understand that translations are specified by a distance and direction (using a vector).

Recognise that enlargements preserve angle but not length.

Understand that distances and angles are preserved under rotations, reflections and translations so that any shape is congruent to its image.

Understand that similar shapes are enlargements of each other and angles are preserved.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

**COMMON MISCONCEPTIONS**

Students often use the term ‘transformation’ when describing transformations instead of the required information.

Lines parallel to the coordinate axes often get confused.

**NOTES**

Emphasise the need to describe the transformations fully, and if asked to describe a ‘single’ transformation students should not include two types.

Find the centre of rotation, by trial and error and by using tracing paper. Include centres on or inside shapes.

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| **6. Quadratics including the formula and iteration** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should

Understand the ≥ and ≤ symbols.

Be able to substitute into, solve and rearrange linear equations.

Be able to factorise quadratic expressions

**KEYWORDS**

Quadratic, solution, root, linear, solve, completing the square, factorise, rearrange, surd, function, solve, circle,

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Factorise quadratic expressions in the form *ax*2 + *bx* + *c*;
* Solve quadratic equations by factorisation and completing the square;
* Solve quadratic equations that need rearranging;
* Set up and solve quadratic equations;
* Solve quadratic equations by using the quadratic formula;
* Interpret the solution in the context of the problem;
* Show that the solution to an equation lies between two values
* Solve quadratic (and cubic) equations using an iterative process
* Use iteration with simple converging sequences

**POSSIBLE SUCCESS CRITERIA**

Solve 3*x*2 + 4 = 100.

Know that the quadratic formula can be used to solve all quadratic equations, and often provides a more efficient method than factorising or completing the square.

Have an understanding of solutions that can be written in surd form.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

**COMMON MISCONCEPTIONS**

Using the formula involving negatives can result in incorrect answers.

If students are using calculators for the quadratic formula, they can come to rely on them and miss the fact that some solutions can be left in surd form.

**NOTES**

Remind students to use brackets for negative numbers when using a calculator, and remind them of the importance of knowing when to leave answers in surd form.

The quadratic formula must now be known; it will not be given in the exam paper.

Reinforce the fact that some problems may produce one inappropriate solution which can be ignored.

Clear presentation of working out is essential.

Link with graphical representations.

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| **7. Simultaneous Equations** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to:

Substitute into, solve and rearrange linear equations.

Factorise simple quadratic expressions.

Show inequalities on number lines;

Write down whole number values that satisfy an inequality;

Solve simple linear inequalities in one variable, and represent the solution set on a number line;

**KEYWORDS**

Quadratic, solution, root, linear, solve, simultaneous, inequality, completing the square, factorise, rearrange, surd, function, solve, circle, sets, union, intersection

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Find the exact solutions of two simultaneous equations in two unknowns;
* Use elimination or substitution to solve simultaneous equations;
* Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:
* linear / linear, including where both need multiplying;
* linear / quadratic;
* linear / *x*2 + *y*2 = *r*2;
* Set up and solve a pair of linear simultaneous equations in two variables, including to represent a situation;
* Interpret the solution in the context of the problem;
* Solve two linear inequalities in *x*, find the solution sets and compare them to see which value of *x* satisfies both solve linear inequalities in two variables algebraically;
* Use the correct notation to show inclusive and exclusive inequalities.

**POSSIBLE SUCCESS CRITERIA**

Have an understanding of solutions that can be written in surd form.

Use inequality symbols to compare numbers.

Given a list of numbers, represent them on a number line using the correct notation.

Solve equations involving inequalities.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Problems that require students to set up and solve a pair of simultaneous equations in a
real-life context, such as 2 adult tickets and 1 child ticket cost £28, and 1 adult ticket and 3 child tickets cost £34. How much does 1 adult ticket cost?

Problems that require student to justify why certain values in a solution can be ignored.

**COMMON MISCONCEPTIONS**

When solving inequalities students often state their final answer as a number quantity, and exclude the inequality or change it to =.

Some students believe that –6 is greater than –3.

**NOTES**

Link with graphical representations.

Emphasise the importance of leaving their answer as an inequality (and not changing it to =).

Students can leave their answers in fractional form where appropriate.

Ensure that correct language is used to avoid reinforcing misconceptions: for example, 0.15 should never be read as ‘zero point fifteen’, and 5 > 3 should be read as ‘five is greater than 3’, not ‘5 is bigger than 3’.

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| **8. Probability** | **Teaching Time**9–11 hours |

**MUCH OF THIS UNIT IS BUILT UPON PROBABILITY TREES, TWO WAY TABLES AND VENN DIAGRAMS FROM THE CROSSOVER UNITS – BUT THE TOPIC NEEDS TO BE DEVELOPED FURTHER TO INCLUDE OTHER AREAS SUCH AS RATIO, PERCENTAGES AND FORMING AND SOLVING EQUATIONS IN THIS CONTEXT**

**PRIOR KNOWLEDGE**

**PRIOR KNOWLEDGE**

Students should :

Understand that a probability is a number between 0 and 1, and distinguish between events which are impossible, unlikely, even chance, likely, and certain to occur.

Be able to mark events and/or probabilities on a probability scale of 0 to 1.

Know how to add and multiply fractions and decimals and be able to write probabilities using fractions, percentages or decimals;

Understand and use experimental and theoretical measures of probability, including relative frequency to include outcomes using dice, spinners, coins, etc;

Estimate the number of times an event will occur, given the probability and the number of trials;

Be able to Find the probability of successive events, such as several throws of a single dice;

List all outcomes for single events, and combined events, systematically;

Draw sample space diagrams and use them for adding simple probabilities;

Know that the sum of the probabilities of all outcomes is 1;

Use 1 – *p* as the probability of an event not occurring where *p* is the probability of the event occurring;

Work out probabilities from Venn diagrams to represent real-life situations and also ‘abstract’ sets of numbers/values and use union and intersection notation;

Be able to work with Venn diagrams, probability trees and two-way tables.

**KEYWORDS**

Probability, mutually exclusive, conditional, tree diagrams, sample space, outcomes, theoretical, relative frequency, Venn diagram, fairness, experimental

**OBJECTIVES**

By the end of the unit, students should be able to:

* Find a missing probability from a list or two-way table, including algebraic terms;
* Understand conditional probabilities and decide if two events are independent;
* Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcomes including algebraic terms;
* Understand selection with or without replacement;
* Calculate the probability of independent and dependent combined events including algebraic terms
* Use a two-way table to calculate conditional probability;
* Use a tree diagram to calculate conditional probability;
* Use a Venn diagram to calculate conditional probability;
* Compare experimental data and theoretical probabilities;
* Compare relative frequencies from samples of different sizes.

**POSSIBLE SUCCESS CRITERIA**

Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Students should be given the opportunity to justify the probability of events happening or not happening in real-life and abstract contexts.

**COMMON MISCONCEPTIONS**

Probability without replacement is best illustrated visually and by initially working out probability ‘with’ replacement.

Not using fractions or decimals when working with probability trees.

**NOTES**

Encourage students to work ‘across’ the branches, working out the probability of each successive event. The probability of the combinations of outcomes should = 1.

Use problems involving ratio and percentage, similar to:

* A bag contains balls in the ratio 2 : 3 : 4. A ball is taken at random. Work out the probability that the ball will be … ;
* In a group of students 55% are boys, 65% prefer to watch film *A*, 10% are girls who prefer to watch film *B*. One student picked at random. Find the probability that this is a boy who prefers to watch film *A* (P6).

Emphasise that, were an experiment repeated, it will usually lead to different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics.

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| **9. Direct and inverse proportion** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should:

Be able to draw linear and quadratic graphs.

Have knowledge of writing statements of direct proportion and forming an equation to find values.

**KEYWORDS**

Equation, direct, indirect, proportion, constant of proportionality

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Identify direct proportion from a table of values, by comparing ratios of values;
* Recognise when values are in direct and inverse proportion by reference to the graph form;
* Identify direct proportion from a table of values, by comparing ratios of values, for
*x* squared and *x* cubed relationships;
* Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity;
* Set up and use equations to solve word and other problems involving direct proportion;
* Relate algebraic solutions to graphical representation of the equations;
* Use *y* = *kx* to solve direct proportion problems, including questions where students find *k*, and then use *k* to find another value;
* Solve problems involving inverse proportion using graphs by plotting and reading values from graphs;
* Solve problems involving inverse proportionality;
* Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.
* Calculate an unknown quantity from quantities that vary in direct or inverse proportion;
* Recognise when values are in inverse proportion by reference to the graph form;

**POSSIBLE SUCCESS CRITERIA**

Understand that when two quantities are in direct proportion, the ratio between them remains constant.

Know the symbol for ‘is proportional to’.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Justify and infer relationships in real-life scenarios to direct and inverse proportion such as ice cream sales and sunshine.

**COMMON MISCONCEPTIONS**

Direct and inverse proportion can get mixed up.

**NOTES**

Consider using science contexts for problems involving inverse proportionality, e.g. volume of gas inversely proportional to the pressure or frequency is inversely proportional to wavelength.

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| **10. Similarity in 2D and 3D** | **Teaching Time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to :

Recognise and enlarge shapes and calculate scale factors.

Calculate area and volume in various metric measures.

Measure lines and angles, and use compasses, ruler and protractor to construct standard constructions.

Identify shapes which are similar; including all circles or all regular polygons with equal number of sides;

Understand similarity of triangles and of other plane shapes, use this to make geometric inferences, and solve angle problems using similarity;

Understand the effect of enlargement on angles and perimeter of shapes;

Know that scale diagrams, including bearings and maps are ‘similar’ to the real-life examples.

Solve problems to find missing lengths in similar shapes;

**KEYWORDS**

Similar, length, area, volume, scale factor, square, cube, roots.

**OBJECTIVES**

By the end of the unit, students should be able to:

* Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
* Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
* Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fraction scale factors;
* Write the lengths, areas and volumes of two shapes as ratios in their simplest form;
* Find missing lengths, areas and volumes in similar 3D solids;
* Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids;
* Use the relationship between enlargement and areas and volumes of simple shapes and solids;
* Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

**POSSIBLE SUCCESS CRITERIA**

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not.

Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Multi-step questions which require calculating missing lengths of similar shapes prior to calculating area of the shape, or using this information in trigonometry or Pythagoras problems.

**COMMON MISCONCEPTIONS**

Students commonly use the same scale factor for length, area and volume.

**NOTES**

Encourage students to model consider what happens to the area when a 1 cm square is enlarged by a scale factor of 3.

Ensure that examples involving given volumes are used, requiring the cube root being calculated to find the length scale factor.

Make links between similarity and trigonometric ratios.

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| **11. Sine and cosine rules, *ab* sin *C*, trigonometry and Pythagoras’ Theorem in 3D, trigonometric graphs** |

**PRIOR KNOWLEDGE**

Students should be able to

Use axes and coordinates to specify points in all four quadrants.

Recall and apply Pythagoras’ Theorem and trigonometric ratios.

Substitute into formulae.

**KEYWORDS**

Axes, coordinates, sine, cosine, tan, angle, graph, transformations, side, angle, inverse, square root, 2D, 3D, diagonal, plane, cuboid

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| **11a. Graphs of trigonometric functions**  | **Teaching time**TBC |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Recognise, sketch and interpret graphs of the trigonometric functions (in degrees)
*y* = sin *x*, *y* = cos *x* and *y* = tan *x* for angles of any size.
* Know the exact values of sin *θ* and cos *θ* for *θ* = 0°, 30°, 45° , 60° and 90° and exact value of tan *θ* for *θ* = 0°, 30°, 45° and 60° and find them from graphs.
* Apply to the graph of *y* = f(*x*) the transformations *y* = –f(*x*), *y* = f(–*x*) for sine, cosine and tan functions f(*x*).
* Apply to the graph of *y* = f(*x*) the transformations *y* = f(*x*) + *a*, *y* = f(*x* + *a*)
for sine, cosine and tan functions f(*x*).

**POSSIBLE SUCCESS CRITERIA**

Match the characteristic shape of the graphs to their functions and transformations.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Match a given list of events/processes with their graph.

Calculate and justify specific coordinates on a transformation of a trigonometric function.

**NOTES**

Translations and reflections of functions are included in this specification, but not rotations or stretches.

This work could be supported by the used of graphical calculators or suitable ICT.

Students need to recall the above exact values for sin, cos and tan.

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| **11b. Further trigonometry**  | **Teaching time**TBC |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Know and apply Area = *ab* sin *C* to calculate the area, sides or angles of any triangle.
* Know the sine and cosine rules, and use to solve 2D problems (including involving bearings).
* Use the sine and cosine rules to solve 3D problems.
* Understand the language of planes, and recognise the diagonals of a cuboid.
* Solve geometrical problems on coordinate axes.
* Give an answer to the use of Pythagoras’ Theorem in surd form;
* Understand, recall and use trigonometric relationships and Pythagoras’ Theorem in right-angled triangles, and use these to solve problems in 3D configurations.
* Calculate the length of a diagonal of a cuboid.
	+ Find the angle between a line and a plane.

**POSSIBLE SUCCESS CRITERIA**

Find the area of a segment of a circle given the radius and length of the chord.

Justify when to use the cosine rule, sine rule, Pythagoras’ Theorem or normal trigonometric ratios to solve problems.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Triangles formed in a semi-circle can provide links with other areas of mathematics.

**COMMON MISCONCEPTIONS**

Not using the correct rule, or attempting to use ‘normal trig’ in non-right-angled triangles.

When finding angles students will be unable to rearrange the cosine rule or fail to find the inverse of cos *θ*.

**NOTES**

The cosine rule is used when we have SAS and used to find the side opposite the ‘included’ angle or when we have SSS to find an angle.

Ensure that finding angles with ‘normal trig’ is refreshed prior to this topic.

Students may find it useful to be reminded of simple geometrical facts, i.e. the shortest side is always opposite the shortest angle in a triangle.

The sine and cosine rules and general formula for the area of a triangle are not given on the formulae sheet.

In multi-step questions emphasise the importance of not rounding prematurely and using exact values where appropriate.

Whilst 3D coordinates are not included in the programme of study, they provide a visual introduction to trigonometry in 3D.

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| **12. Sampling, cumulative frequency, box plots and histograms** |

**PRIOR KNOWLEDGE**

Students should understand the different types of data: discrete/continuous.

Students should have experience of inequality notation.

Students should be able to multiply a fraction by a number.

Students should understand the data handling cycle, specifically:

* Specify the problem and plan:
* decide what data to collect and what analysis is needed;
* understand primary and secondary data sources;
* consider fairness;

Understand what is meant by a sample and a population;

Understand how different sample sizes may affect the reliability of conclusions drawn;

Identify possible sources of bias and plan to minimise it;

Write questions to eliminate bias, and understand how the timing and location of a survey can ensure a sample is representative

**KEYWORDS**

Sample, population, fraction, decimal, percentage, bias, stratified sample, random, cumulative frequency, box plot, histogram, frequency density, frequency, mean, median, mode, range, lower quartile, upper quartile, interquartile range, spread, comparison, outlier

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| **12a. Sampling** | **Teaching time**TBC |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Compare relative frequencies from samples of different sizes. (Capture-recapture)

**POSSIBLE SUCCESS CRITERIA**

A farmer catches 120 rabbits on Monday and he puts a tag on each rabbit. He then lets the rabbits run away.

On Tuesday the farmer catches 70 rabbits.15 of these rabbits have a tag on them.

Work out an estimate for the total number of rabbits on the farm.

**NOTES**

Different types of sampling such as simple, random, stratified sampling etc are not explicitly included in the GCSE but worth covering to look at capture-recapture in the context of samples and populations.

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| **12b. Cumulative frequency & box plots** | **Teaching time**TBC |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Use statistics found in all graphs/charts in this unit to describe a population;
* Know the appropriate uses of cumulative frequency diagrams;
* Construct and interpret cumulative frequency tables;
* Construct and interpret cumulative frequency graphs/diagrams and from the graph:
* estimate frequency greater/less than a given value;
* find the median and quartile values and interquartile range;
* Compare the mean, mode and range of two distributions, or median and interquartile range, as appropriate;
* Interpret box plots to find median, quartiles, range and interquartile range and draw conclusions;
* Produce box plots from raw data and when given quartiles, median and identify any outliers;

**POSSIBLE SUCCESS CRITERIA**

Construct cumulative frequency graphs and box plots from frequency tables.

Compare two data sets and justify their comparisons based on measures extracted from their diagrams where appropriate in terms of the context of the data.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Interpret two or more data sets from box plots and relate the key measures in the context of the data.

Given the size of a sample and its box plot calculate the proportion above/below a specified value.

**COMMON MISCONCEPTIONS**

Labelling axes incorrectly in terms of the scales, and also using ‘Frequency’ instead of ‘Frequency Density’ or ‘Cumulative Frequency’.

Students often confuse the methods involved with cumulative frequency, estimating the mean and histograms when dealing with data tables.

**NOTES**

Ensure that axes are clearly labelled.

As a way to introduce measures of spread, it may be useful to find mode, median, range and interquartile range from stem and leaf diagrams (including back-to-back) to compare two data sets.

As an extension, use the formula for identifying an outlier, (i.e. if data point is below
LQ – 1.5 × IQR or above UQ + 1.5 × IQR, it is an outlier). Get them to identify outliers in the data, and give bounds for data.

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| **12c. Histograms**  | **Teaching time**TBC |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Know the appropriate uses of histograms;
* Produce histograms with equal class intervals:
* Estimate the median from a histogram with equal class width or any other information, such as the number of people in a given interval;
* Construct and interpret histograms from class intervals with unequal width;
* Use and understand frequency density;
* From histograms:
* complete a grouped frequency table;
* understand and define frequency density;
* Estimate the mean from a histogram;
* Estimate the median from a histogram with unequal class widths or any other information from a histogram, such as the number of people in a given interval.

**POSSIBLE SUCCESS CRITERIA**

Construct histograms from frequency tables.

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**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

**COMMON MISCONCEPTIONS**

Labelling axes incorrectly in terms of the scales, and also using ‘Frequency’ instead of ‘Frequency Density’ or ‘Cumulative Frequency’.

Students often confuse the methods involved with cumulative frequency, estimating the mean and histograms when dealing with data tables.

**NOTES**

Ensure that axes are clearly labelled.

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| **13. Graphs** |  |

**PRIOR KNOWLEDGE**

Students should be able to

Solve quadratics and linear equations and simultaneous equations algebraically.

Draw linear and quadratic graphs.

Calculate the gradient of a linear function between two points.

Write statements of direct proportion and forming an equation to find values.

Recognise and sketch graphs of the reciprocal function  with *x* ≠ 0

**KEYWORDS**

Sketch, estimate, quadratic, cubic, function, factorising, simultaneous equation, graphical, algebraic, time, velocity, transformations, cubic, transformation, Reciprocal, linear, gradient, quadratic, exponential, functions, area, rate of change, distance,

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| **13a. Using graphs of circles, cubes and quadratics** | **Teaching time**TBC |

**OBJECTIVES**

By the end of the unit, students should be able to:

* Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots, *y*-intercept and turning point by completing the square;
* Be able to identify from a graph if a quadratic equation has any real roots;
* Find approximate solutions to quadratic equations using a graph;
* Sketch a graph of a quadratic function and a linear function, identifying intersection points;
* Sketch graphs of simple cubic functions, given as three linear expressions;
* Solve simultaneous equations graphically:
* find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach;
* find graphically the intersection points of a given straight line with a circle;
* solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem;
* Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;
* Represent the solution set for inequalities using set notation, i.e. curly brackets and ‘is an element of’ notation;
* for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of *x*² – 3*x* – 10 < 0 as {*x*: –3 < *x* < 5};
* Solve linear inequalities in two variables graphically;
* Show the solution set of several inequalities in two variables on a graph;

**POSSIBLE SUCCESS CRITERIA**

Expand *x*(*x* – 1)(*x* + 2).

Expand (*x* – 1)3.

Expand (*x* + 1)(*x* + 2)(*x* – 1).

Sketch *y* = (*x* + 1)2(*x* – 2).

Interpret a pair of simultaneous equations as a pair of straight lines and their solution as the point of intersection.

Be able to state the solution set of *x*² – 3*x* – 10 < 0 as {*x*: *x* < -3}  {*x*: *x* > 5}.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Match equations to their graphs and to real-life scenarios.

“Show that”-type questions will allow students to show a logical and clear chain of reasoning.

**COMMON MISCONCEPTIONS**

When estimating values from a graph, it is important that students understand it is an ‘estimate’.

It is important to stress that when expanding quadratics, the *x* terms are also collected together.

Quadratics involving negatives sometimes cause numerical errors.

**NOTES**

You may want to extend the students to include expansions of more than three linear expressions.

Practise expanding ‘double brackets’ with all combinations of positives and negatives.

Set notation is a new topic.

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|  **13b. Gradient and area under graphs**  | **Teaching time**TBC |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Interpret graphs of the reciprocal function  with *x* ≠ 0
* State the value of *x* for which the equation  with *x* ≠ 0 is not defined;
* Recognise, sketch and interpret graphs of exponential functions *y* = *kx* for positive values of *k* and integer values of *x*;
* Use calculators to explore exponential growth and decay;
* Set up, solve and interpret the answers in growth and decay problems;
* Estimate area under a quadratic or other graph by dividing it into trapezia;
* Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
* Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
* for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
* for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
* Interpret the gradient of a linear or non-linear graph in financial contexts;
* Interpret the area under a linear or non-linear graph in real-life contexts;
* Interpret the rate of change of graphs of containers filling and emptying;
* Interpret the rate of change of unit price in price graphs.

**POSSIBLE SUCCESS CRITERIA**

Explain why you cannot find the area under a reciprocal or tan graph.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Interpreting many of these graphs in relation to their specific contexts.

**COMMON MISCONCEPTIONS**

The effects of transforming functions is often confused.

**NOTES**

Formal function notation along with inverse and composite functions will have been encountered but are topics that students may need to be reminded about.

Translations and reflections of functions are included in this specification, but not rotations or stretches.

Financial contexts could include percentage or growth rate.

When interpreting rates of change with graphs of containers filling and emptying, a steeper gradient means a faster rate of change.

When interpreting rates of change of unit price in price graphs, a steeper graph means larger unit price.

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| **14. Circle geometry**  | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should

Have practical experience of drawing circles with compasses.

Be able to recall the words, centre, radius, diameter and circumference.

Be able to recall the relationship of the gradient between two perpendicular lines.

Be able to find the equation of the straight line, given a gradient and a coordinate.

**KEYWORDS**

Radius, centre, tangent, circumference, diameter, gradient, perpendicular, reciprocal, coordinate, equation, substitution, chord, triangle, isosceles, angles, degrees, cyclic quadrilateral, alternate, segment, semicircle, arc, theorem

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines;
* Find the equation of a tangent to a circle at a given point, by:
* finding the gradient of the radius that meets the circle at that point (circles all centre the origin);
* finding the gradient of the tangent perpendicular to it;
* using the given point;
* Recognise and construct the graph of a circle using *x*2 + *y*2 = *r*2 for radius *r* centred at the origin of coordinates.

**POSSIBLE SUCCESS CRITERIA**

Find the gradient of a radius of a circle drawn on a coordinate grid and relate this to the gradient of the tangent.

Justify the relationship between the gradient of a tangent and the radius.

Produce an equation of a line given a gradient and a coordinate.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Justify if a straight-line graph would pass through a circle drawn on a coordinate grid.

**COMMON MISCONCEPTIONS**

Students find it difficult working with negative reciprocals of fractions and negative fractions.

**NOTES**

Work with positive gradients of radii initially and review reciprocals prior to starting this topic.

It is useful to start this topic through visual proofs, working out the gradient of the radius and the tangent, before discussing the relationship.

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| **15. Circle theorems**  | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should

Have practical experience of drawing circles with compasses.

Be able to recall the words, centre, radius, diameter and circumference.

**KEYWORDS**

Radius, centre, tangent, circumference, diameter, perpendicular, chord, triangle, isosceles, angles, degrees, cyclic quadrilateral, alternate, segment, semicircle, arc, theorem

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Recall the definition of a circle and identify (name) and draw parts of a circle, including sector, tangent, chord, segment;
* Prove and use the facts that:
* the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference; the angle in a semicircle is a right angle;
* the perpendicular from the centre of a circle to a chord bisects the chord;
* angles in the same segment are equal;
* alternate segment theorem;
* opposite angles of a cyclic quadrilateral sum to 180°;
* Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point;
* Find and give reasons for missing angles on diagrams using:
* circle theorems;
* isosceles triangles (radius properties) in circles;
* the fact that the angle between a tangent and radius is 90°;
* the fact that tangents from an external point are equal in length.

**POSSIBLE SUCCESS CRITERIA**

Justify clearly missing angles on diagrams using the various circle theorems.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Problems that involve a clear chain of reasoning and provide counter-arguments to statements.

Can be linked to other areas of mathematics by incorporating trigonometry and Pythagoras’ Theorem.

**COMMON MISCONCEPTIONS**

Much of the confusion arises from mixing up the diameter and the radius.

**NOTES**

Reasoning needs to be carefully constructed and correct notation should be used throughout.Students should label any diagrams clearly, as this will assist them; particular emphasis should be made on labelling any radii in the first instance.

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| **16. Algebraic fractions**  | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to

Simplify surds.

Use negative numbers with all four operations.

Recall and use the hierarchy of operations.

**KEYWORDS**

Rationalise, denominator, surd, rational, irrational, fraction, equation, rearrange, subject, proof, function notation, inverse, evaluate

**OBJECTIVES**

By the end of the unit, students should be able to:

* Simplify algebraic fractions;
* Multiply and divide algebraic fractions;
* Solve linear and quadratic equations arising from algebraic fractions.

**POSSIBLE SUCCESS CRITERIA**

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

**COMMON MISCONCEPTIONS**

When simplifying involving factors, students often use the ‘first’ factor that they find and not the LCM.

**NOTES**

Practice factorisation where the factor may involve more than one variable.

Emphasise that, by using the LCM for the denominator, the algebraic manipulation is easier.

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| **17. Functions** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to

Simplify surds.

Use negative numbers with all four operations.

Recall and use the hierarchy of operations.

**KEYWORDS**

Rationalise, denominator, surd, rational, irrational, fraction, equation, rearrange, subject, proof, function notation, inverse, evaluate

**OBJECTIVES**

By the end of the unit, students should be able to:

* Write a ratio as a linear function;
* Use function notation;
* Find f(*x*) + g(*x*) and f(*x*) – g(*x*), 2f(*x*), f(3*x*) etc algebraically;
* Find the inverse of a linear function;
* Know that f –1(*x*) refers to the inverse function;
* For two functions f(*x*) and g(*x*), find gf(*x*).
* Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of f(*x*) + *a*, or f(*x* – *a*):
* apply to the graph of *y* = f(*x*) the transformations *y* = –f(*x*), *y* = f(–*x*) for linear, quadratic, cubic functions;
* apply to the graph of y = f(*x*) the transformations *y* = f(*x*) + *a*, *y* = f(*x* + *a*)
for linear, quadratic, cubic functions;

**POSSIBLE SUCCESS CRITERIA**

Given a function, evaluate f(2).

When g(*x*) = 3 – 2*x*, find g–1 (*x*).

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

**COMMON MISCONCEPTIONS**

**NOTES**

|  |  |
| --- | --- |
| **18. Algebraic Proof** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to

Know the difference between odd and even numbers

Expand expressions including those with single, double and triple brackets.

Solve linear and quadratic equations.

Explain the difference between equations, expressions and identities.

**KEYWORDS**

Expression, proof, equation, identity, expand, prove, same, counter-example.

**OBJECTIVES**

By the end of the unit, students should be able to:

* Solve ‘Show that’ and proof questions using consecutive integers (*n*, *n* + 1), squares *a*2, *b*2, even numbers 2*n*, odd numbers 2*n* +1 including those that involve products;
* Solve ‘Show that’ and proof questions in context including (but not limited to) area, perimeter and volume;

**POSSIBLE SUCCESS CRITERIA**

Explain why any odd number can be written as 2n + 1 or 2n – 1

Prove that the product of two consecutive even numbers is a multiple of 4

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Formal proof is an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics.

**COMMON MISCONCEPTIONS**

**NOTES**

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| **19. Congruence and geometric proof** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students should be able to

Use the basic congruence criteria for triangles (SSS, SAS, ASA and RHS);

Understand congruence, as two shapes that are the same size and shape and visually identify shapes which are congruent;

Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor;

**KEYWORDS**

Congruence, side, angle, shape, volume, length, area, volume, scale factor, enlargement, proof,

**OBJECTIVES**

By the end of the unit, students should be able to:

* Understand and derive the proof that the angle sum of a triangle is 180°,
* Use formal geometric proof for the similarity of two given triangles;
* Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles and other shapes using formal arguments,
* Solve angle problems by first proving congruence;

**POSSIBLE SUCCESS CRITERIA**

Understand that enlargement does not have the same effect on area and volume.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Formal proof is an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics.

**COMMON MISCONCEPTIONS**

Students commonly use the same scale factor for length, area and volume.

**NOTES**

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| **20. Vectors** | **Teaching time**TBC |

**PRIOR KNOWLEDGE**

Students will have used vectors to describe translations and will have knowledge of Pythagoras’ Theorem and the properties of triangles and quadrilaterals. They should also have done some column vector arithmetic.

**KEYWORDS**

Vector, direction, magnitude, scalar, multiple, parallel, collinear, proof, ratio, column vector

**OBJECTIVES**

By the end of the unit, students should be able to:

* Understand and use vector notation, including column notation, and understand and interpret vectors as displacement in the plane with an associated direction.
* Understand that 2**a** is parallel to **a** and twice its length, and that **a** is parallel to –**a** in the opposite direction.
* Represent vectors, combinations of vectors and scalar multiples in the plane pictorially.
* Calculate the sum of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms).
* Find the length of a vector using Pythagoras’ Theorem.
* Calculate the resultant of two vectors.
* Solve geometric problems in 2D where vectors are divided in a given ratio.
* Produce geometrical proofs to prove points are collinear and vectors/lines are parallel.

**POSSIBLE SUCCESS CRITERIA**

Add and subtract vectors algebraically and use column vectors. Solve geometric problems and produce proofs.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

“Show that”-type questions are an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics, in particular algebra.

Find the area of a parallelogram defined by given vectors.

**COMMON MISCONCEPTIONS**

Students find it difficult to understand that parallel vectors are equal as they are in different locations in the plane.

**NOTES**

Students find manipulation of column vectors relatively easy compared to pictorial and algebraic manipulation methods – encourage them to draw any vectors they calculate on the picture.

Geometry of a hexagon provides a good source of parallel, reverse and multiples of vectors.

Remind students to underline vectors or use an arrow above them, or they will be regarded as just lengths.

Extend geometric proofs by showing that the medians of a triangle intersect at a single point.

3D vectors or **i**, **j** and **k** notation can be introduced and further extension work can be found in old GCE Mechanics 1 textbooks.